COEFFICENT INVERSE PROBLEM FOR SOLUTE TRANSPORT IN A CYLINDRICAL NON-HOMOGENEOUS POROUS MEDIUM

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A lot number of investigation is devoted to the solute transport in macroscopic non-homogeneous porous media [1, 2].

In several works the sape of the medium is defined, such as cylinder, sphere, plate-layered medium etc. In one of them two coaxial cylinders are used, the intrinsic one ha relatively higher filtration-capacity properties (macrapore), and surrounding one-relatively less properties (micropore). As a rule two approaches are used: diffusional and kinetic.

In this work we solve an inverse problem to determine such a values of the mass-transfer coefficient in the kinetic equation that two approaches give close results.

In diffusional approach solute transport is described by

$$\theta_m \frac{\partial c_m}{\partial t} + \theta_{im} \frac{\partial c_{im}}{\partial t} = \theta_m D_m \frac{\partial^2 c_m}{\partial x^2} - \theta_m v_m \frac{\partial c_m}{\partial x}, \qquad (1)$$

where c_m - volumetric concentration of solute, c_{im} - average volumetric concentration of solute in a micropore, D_m- hydrodynamic dispersion, v_m - average velocity of solute, c_a - local concentration of solute in a macropore, t - time, x- spatial variable, θ_m, θ_{im} - porosity of the macropor and micropor, respectively;

$$c_{im} = \frac{2}{b^2 - a^2} \int_a^b r c_a(t, x, r) dr, \qquad (2)$$

$$\frac{\partial c_a}{\partial t} = \frac{D_a}{r} \frac{\partial}{\partial r}\left(r \frac{\partial c_a}{\partial r} \right), \quad a < r < b \;, \qquad (3)$$

where D_a is effective diffusion coefficient in the micropore.

In the kinetic approach instead of (2), (3) the following kinetic equation is used

$$\theta_{im}\frac{\partial c_{im}}{\partial t}=\alpha\left(c_m-c_{im}\right),\quad \alpha=\text{const.} \tag{4}$$

To define α in (4) we use the solution of the direct problem in certain points of the medium, received on the basis of diffusional approach. By such a way we prepare, initial data for the inverse problem is the decision of a direct problem

$$c_m\left(t,x_k\right)=z_k\left(t\right),\quad t\in\left[0,T\right],\quad k=1,2,3. \tag{5}$$

To determine α we minimize the following square functional

$$J(\alpha)=\sum_{k=1}^{3}\int_0^T\left[c_m\left(\xi,x_k\right)-z_k\left(\xi\right)\right]^2d\xi. \tag{6}$$

To minimize functional (6) we use Matlab 7.6.0 (R2008a). Values α for which both of the approaches yield close results are determined. Then, by using values of α the corresponding these direct problem is solved and results on the basic of two approaches are compared. It is show that defined values of α provide good convergence of results[3,4].

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